IBM Model 1 and the EM Algorithm

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Lexical Translation

- How to translate a word → look up in dictionary

  Haus — house, building, home, household, shell.

- Multiple translations
  - some more frequent than others
  - for instance: house, and building most common
  - special cases: Haus of a snail is its shell

- Note: In all lectures, we translate from a foreign language into English

Collect Statistics

Look at a parallel corpus (German text along with English translation)

Translation of *Haus*	**Count**
house	8,000
building	1,600
home	200
household	150
shell	50

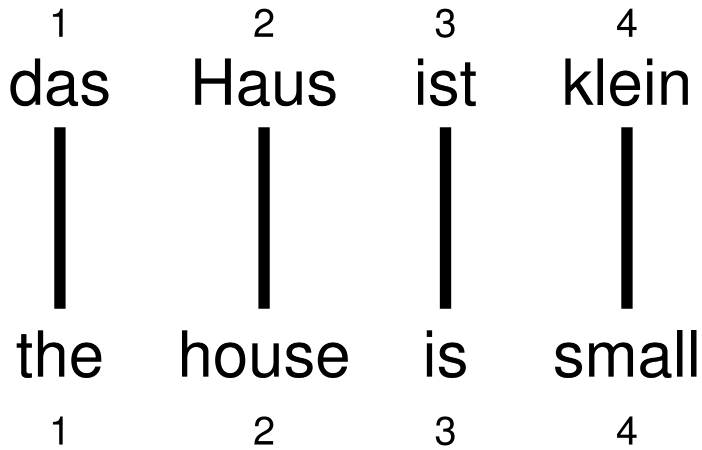
Estimate Translation Probabilities

Maximum likelihood estimation

$$p_f(e) = \begin{cases} 0.8 & \text{if } e = \text{house}, \\ 0.16 & \text{if } e = \text{building}, \\ 0.02 & \text{if } e = \text{home}, \\ 0.015 & \text{if } e = \text{household}, \\ 0.005 & \text{if } e = \text{shell}. \end{cases}$$

Alignment

- In a parallel text (or when we translate), we align words in one language with the words in the other



- Word positions are numbered 1–4

Alignment Function

- Formalizing alignment with an alignment function

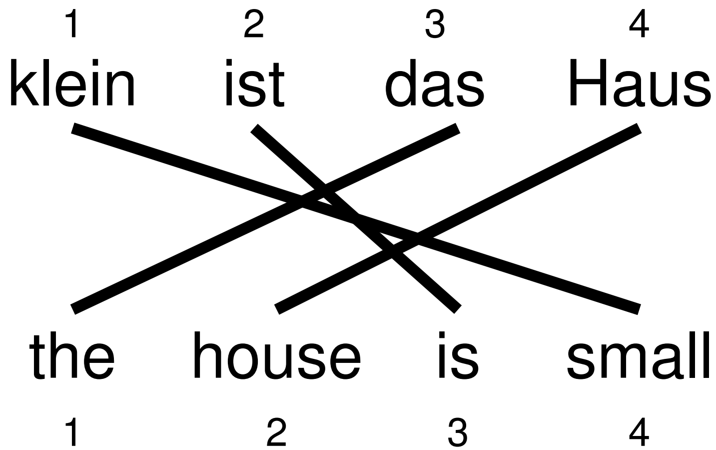
- Mapping an English target word at position i to a German source word at position j with a function $a : i \to j$

- Example

$$a : \{1 \to 1, 2 \to 2, 3 \to 3, 4 \to 4\}$$

Reordering

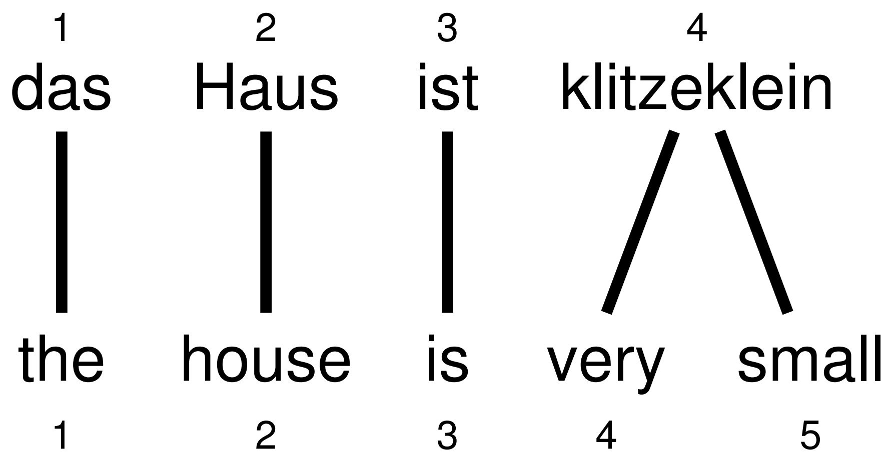
Words may be reordered during translation



$$a : \{1 \rightarrow 3, 2 \rightarrow 4, 3 \rightarrow 2, 4 \rightarrow 1\}$$

One-to-Many Translation

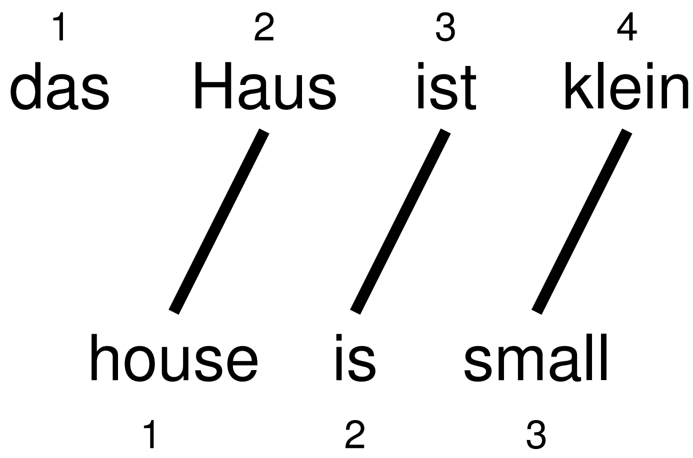
A source word may translate into multiple target words



$$a : \{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 4\}$$

Dropping Words

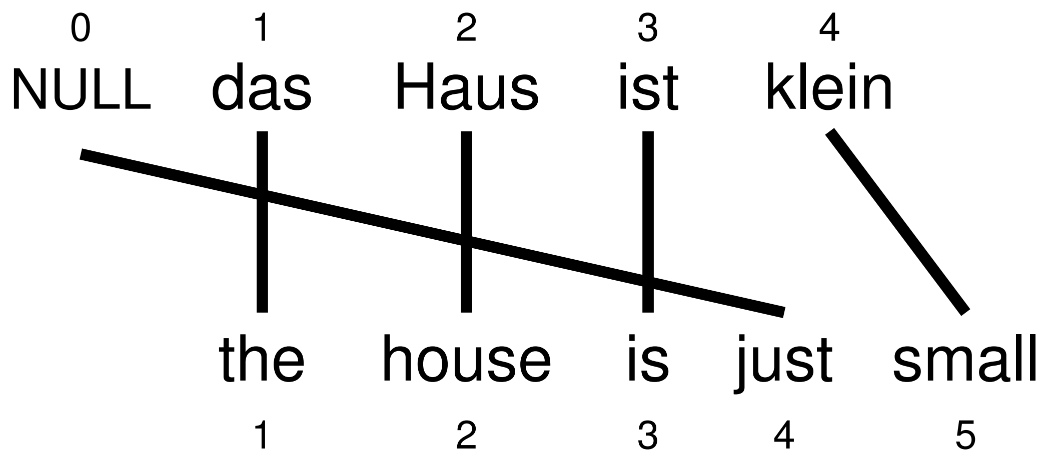
Words may be dropped when translated
(German article das is dropped)



$$a : \{1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 4\}$$

Inserting Words

- Words may be added during translation
  - The English just does not have an equivalent in German
  - We still need to map it to something: special NULL token



$$a : \{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 0, 5 \rightarrow 4\}$$

IBM Model 1

- Generative model: break up translation process into smaller steps
  - IBM Model 1 only uses lexical translation

- Translation probability
  - for a foreign sentence $\mathbf{f} = (f_1, ..., f_{l_f})$ of length l_f
  - to an English sentence $\mathbf{e} = (e_1, ..., e_{l_e})$ of length l_e
  - with an alignment of each English word e_j to a foreign word f_i according to the alignment function $a : j \to i$

$$p(\mathbf{e}, a|\mathbf{f}) = \frac{\epsilon}{(l_f + 1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

  - parameter ϵ is a normalization constant

Example

das

e	$t(e\|f)$
the	0.7
that	0.15
which	0.075
who	0.05
this	0.025

Haus

e	$t(e\|f)$
house	0.8
building	0.16
home	0.02
household	0.015
shell	0.005

ist

e	$t(e\|f)$
is	0.8
's	0.16
exists	0.02
has	0.015
are	0.005

klein

e	$t(e\|f)$
small	0.4
little	0.4
short	0.1
minor	0.06
petty	0.04

$$\begin{aligned} p(e,a|f) &= \frac{\epsilon}{4^3} \times t(\text{the}|\text{das}) \times t(\text{house}|\text{Haus}) \times t(\text{is}|\text{ist}) \times t(\text{small}|\text{klein}) \\ &= \frac{\epsilon}{4^3} \times 0.7 \times 0.8 \times 0.8 \times 0.4 \\ &= 0.0028\epsilon \end{aligned}$$

finding translations

Centauri-Arcturan Parallel Text

1a. ok-voon ororok sprok .
1b. at-voon bichat dat .

2a. ok-drubel ok-voon anok plok sprok .
2b. at-drubel at-voon pippat rrat dat .

3a. erok sprok izok hihok ghirok .
3b. totat dat arrat vat hilat .

4a. ok-voon anok drok brok jok .
4b. at-voon krat pippat sat lat .

5a. wiwok farok izok stok .
5b. totat jjat quat cat .

6a. lalok sprok izok jok stok .
6b. wat dat krat quat cat .

7a. lalok farok ororok lalok sprok izok enemok .
7b. wat jjat bichat wat dat vat eneat .

8a. lalok brok anok plok nok .
8b. iat lat pippat rrat nnat .

9a. wiwok nok izok kantok ok-yurp .
9b. totat nnat quat oloat at-yurp .

10a. lalok mok nok yorok ghirok clok .
10b. wat nnat gat mat bat hilat .

11a. lalok nok crrrok hihok yorok zanzanok .
11b. wat nnat arrat mat zanzanat .

12a. lalok rarok nok izok hihok mok .
12b. wat nnat forat arrat vat gat .

(from Knight (1997): Automating Knowledge Acquisition for Machine Translation)

em algorithm

Learning Lexical Translation Models

- We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus

- ... but we do not have the alignments

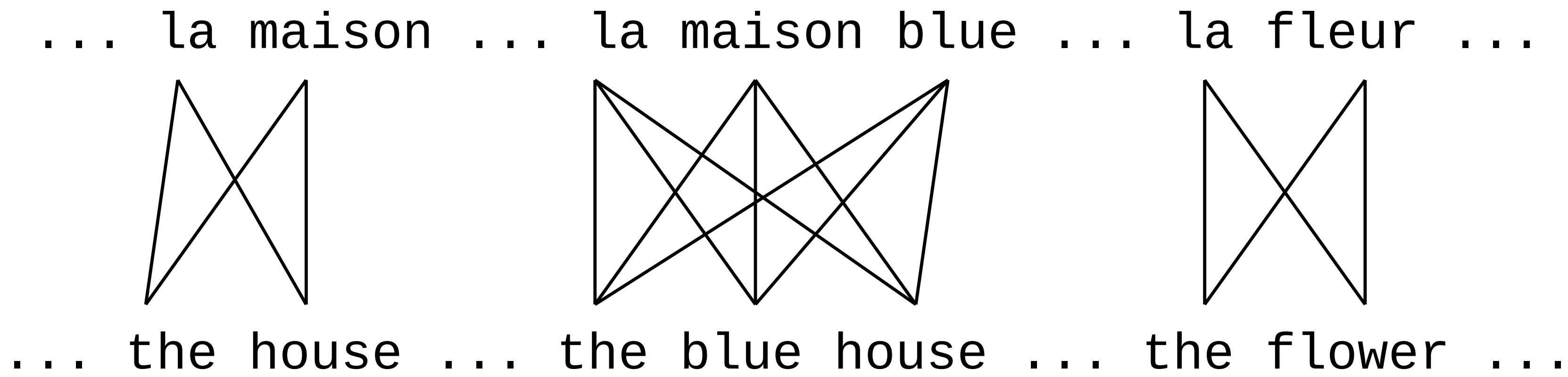
- Chicken and egg problem
  - if we had the *alignments*,
    $\rightarrow$ we could estimate the *parameters* of our generative model
  - if we had the *parameters*,
    $\rightarrow$ we could estimate the *alignments*

EM Algorithm

- Incomplete data
  - if we had *complete data,* would could estimate *model*
  - if we had *model,* we could fill in the *gaps in the data*

- Expectation Maximization (EM) in a nutshell
  1. initialize model parameters (e.g. uniform)
  2. assign probabilities to the missing data
  3. estimate model parameters from completed data
  4. iterate steps 2–3 until convergence

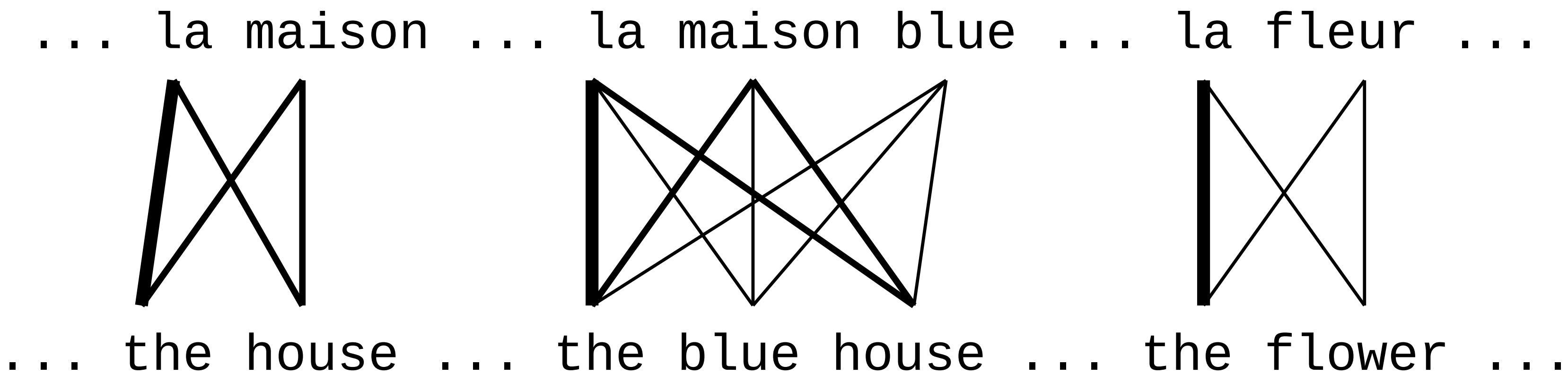
EM Algorithm



- Initial step: all alignments equally likely

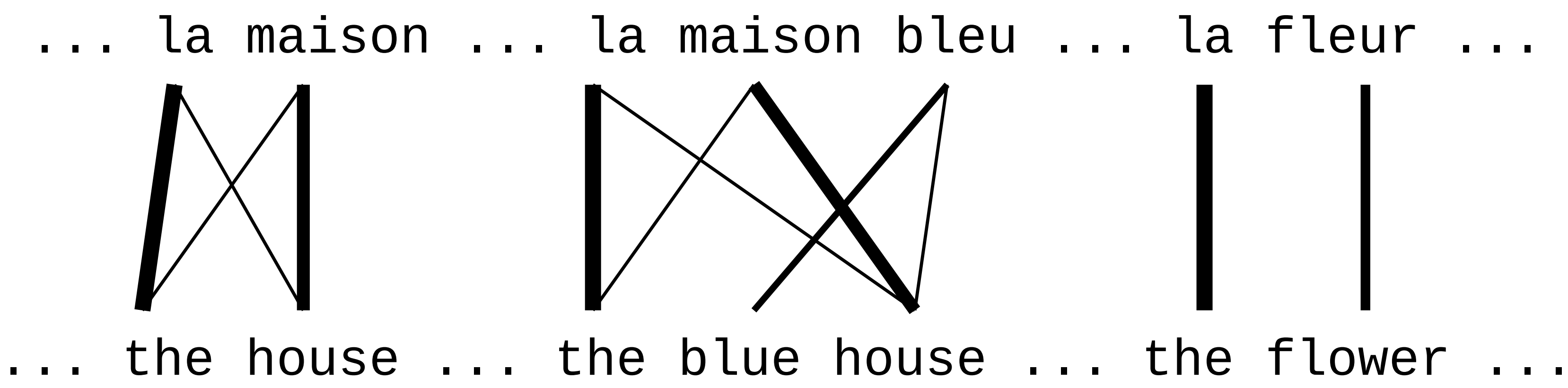
- Model learns that, e.g., la is often aligned with the

EM Algorithm



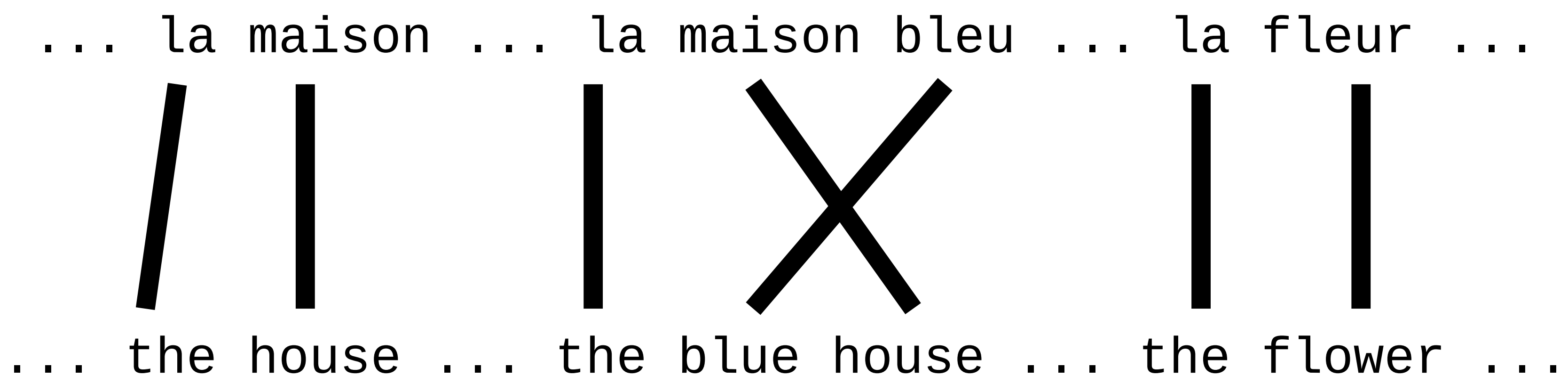
- After one iteration
- Alignments, e.g., between la and the are more likely

EM Algorithm



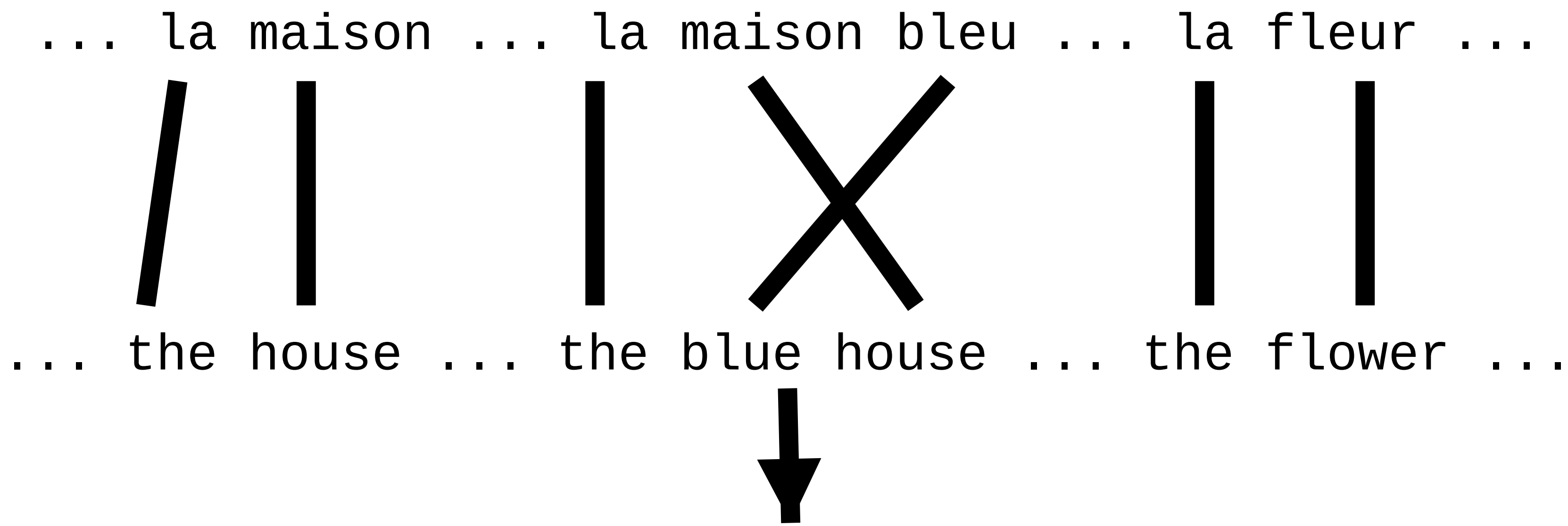
- After another iteration
- It becomes apparent that alignments, e.g., between fleur and flower are more likely (pigeon hole principle)

EM Algorithm



- Convergence
- Inherent hidden structure revealed by EM

EM Algorithm




```
p(la|the) = 0.453
p(le|the) = 0.334
p(maison|house) = 0.876
p(bleu|blue) = 0.563
...
```

- Parameter estimation from the aligned corpus

IBM Model 1 and EM

- EM Algorithm consists of two steps
- Expectation-Step: Apply model to the data
  - parts of the model are hidden (here: alignments)
  - using the model, assign probabilities to possible values
- Maximization-Step: Estimate model from data
  - take assign values as fact
  - collect counts (weighted by probabilities)
  - estimate model from counts
- Iterate these steps until convergence

IBM Model 1 and EM

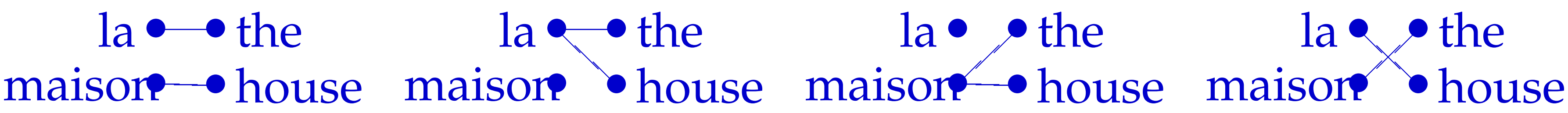
- We need to be able to compute:
  - Expectation-Step: probability of alignments
  - Maximization-Step: count collection

IBM Model 1 and EM

- **Probabilities**

$$p(\text{the}|\text{la}) = 0.7 \qquad p(\text{house}|\text{la}) = 0.05$$
$$p(\text{the}|\text{maison}) = 0.1 \qquad p(\text{house}|\text{maison}) = 0.8$$

- **Alignments**



$$p(\mathbf{e}, a|\mathbf{f}) = 0.56 \qquad p(\mathbf{e}, a|\mathbf{f}) = 0.035 \qquad p(\mathbf{e}, a|\mathbf{f}) = 0.08 \qquad p(\mathbf{e}, a|\mathbf{f}) = 0.005$$

$$p(a|\mathbf{e}, \mathbf{f}) = 0.824 \qquad p(a|\mathbf{e}, \mathbf{f}) = 0.052 \qquad p(a|\mathbf{e}, \mathbf{f}) = 0.118 \qquad p(a|\mathbf{e}, \mathbf{f}) = 0.007$$

- **Counts**

$$c(\text{the}|\text{la}) = 0.824 + 0.052 \qquad c(\text{house}|\text{la}) = 0.052 + 0.007$$
$$c(\text{the}|\text{maison}) = 0.118 + 0.007 \qquad c(\text{house}|\text{maison}) = 0.824 + 0.118$$

IBM Model 1 and EM: Expectation Step

- We need to compute $p(a|\mathbf{e}, \mathbf{f})$
- Applying the chain rule:

$$p(a|\mathbf{e}, \mathbf{f}) = \frac{p(\mathbf{e}, a|\mathbf{f})}{p(\mathbf{e}|\mathbf{f})}$$

- We already have the formula for $p(\mathbf{e}, \mathbf{a}|\mathbf{f})$ (definition of Model 1)

IBM Model 1 and EM: Expectation Step

- We need to compute $p(\mathbf{e}|\mathbf{f})$

$$p(\mathbf{e}|\mathbf{f}) = \sum_a p(\mathbf{e},a|\mathbf{f})$$

$$= \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} p(\mathbf{e},a|\mathbf{f})$$

$$= \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

IBM Model 1 and EM: Expectation Step

$$p(\mathbf{e}|\mathbf{f}) = \sum_{a(1)=0}^{l_f} \dots \sum_{a(l_e)=0}^{l_f} \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

$$= \frac{\epsilon}{(l_f+1)^{l_e}} \sum_{a(1)=0}^{l_f} \dots \sum_{a(l_e)=0}^{l_f} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

$$= \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} \sum_{i=0}^{l_f} t(e_j|f_i)$$

- Note the trick in the last line
  - removes the need for an exponential number of products

→ this makes IBM Model 1 estimation tractable

The Trick

(case $l_e = l_f = 2$)

$$\sum_{a(1)=0}^{2} \sum_{a(2)=0}^{2} = \frac{\epsilon}{3^2} \prod_{j=1}^{2} t(e_j|f_{a(j)}) =$$

$$= t(e_1|f_0)\ t(e_2|f_0) + t(e_1|f_0)\ t(e_2|f_1) + t(e_1|f_0)\ t(e_2|f_2) +$$

$$+ t(e_1|f_1)\ t(e_2|f_0) + t(e_1|f_1)\ t(e_2|f_1) + t(e_1|f_1)\ t(e_2|f_2) +$$

$$+ t(e_1|f_2)\ t(e_2|f_0) + t(e_1|f_2)\ t(e_2|f_1) + t(e_1|f_2)\ t(e_2|f_2) =$$

$$= t(e_1|f_0)\ (t(e_2|f_0) + t(e_2|f_1) + t(e_2|f_2)) +$$

$$+ t(e_1|f_1)\ (t(e_2|f_1) + t(e_2|f_1) + t(e_2|f_2)) +$$

$$+ t(e_1|f_2)\ (t(e_2|f_2) + t(e_2|f_1) + t(e_2|f_2)) =$$

$$= \ (t(e_1|f_0) + t(e_1|f_1) + t(e_1|f_2))\ (t(e_2|f_2) + t(e_2|f_1) + t(e_2|f_2))$$

IBM Model 1 and EM: Expectation Step

- Combine what we have:

$$
\begin{aligned}
p(\mathbf{a}|\mathbf{e},\mathbf{f}) &= p(\mathbf{e},\mathbf{a}|\mathbf{f})/p(\mathbf{e}|\mathbf{f}) \\
&= \frac{\frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})}{\frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} \sum_{i=0}^{l_f} t(e_j|f_i)} \\
&= \prod_{j=1}^{l_e} \frac{t(e_j|f_{a(j)})}{\sum_{i=0}^{l_f} t(e_j|f_i)}
\end{aligned}
$$

IBM Model 1 and EM: Maximization Step

- Now we have to collect counts
- Evidence from a sentence pair **e,f** that word e is a translation of word f:

$$c(e|f;\mathbf{e},\mathbf{f}) = \sum_a p(a|\mathbf{e},\mathbf{f}) \sum_{j=1}^{l_e} \delta(e,e_j)\delta(f,f_{a(j)})$$

- With the same simplication as before:

$$c(e|f;\mathbf{e},\mathbf{f}) = \frac{t(e|f)}{\sum_{i=0}^{l_f} t(e|f_i)} \sum_{j=1}^{l_e} \delta(e,e_j) \sum_{i=0}^{l_f} \delta(f,f_i)$$

IBM Model 1 and EM: Maximization Step

After collecting these counts over a corpus, we can estimate the model:

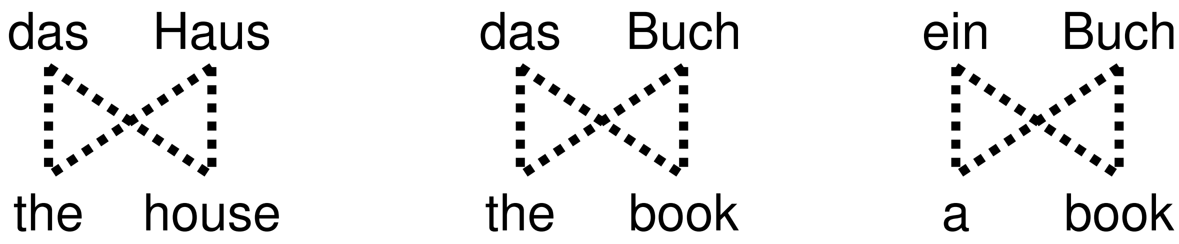
$$t(e|f;\mathbf{e},\mathbf{f}) = \frac{\sum_{(\mathbf{e},\mathbf{f})} c(e|f;\mathbf{e},\mathbf{f}))}{\sum_{e} \sum_{(\mathbf{e},\mathbf{f})} c(e|f;\mathbf{e},\mathbf{f}))}$$

IBM Model 1 and EM: Pseudocode

Input: set of sentence pairs $(\mathbf{e}, \mathbf{f})$
Output: translation prob. $t(e|f)$

```
initialize t(e|f) uniformly
while not converged do
    // initialize
    count(e|f) = 0 for all e, f
    total(f) = 0 for all f
    for all sentence pairs (e,f) do
        // compute normalization
        for all words e in e do
            s-total(e) = 0
            for all words f in f do
                s-total(e) += t(e|f)
            end for
        end for
        // collect counts
        for all words e in e do
            for all words f in f do
                count(e|f) += t(e|f) / s-total(e)
                total(f) += t(e|f) / s-total(e)
            end for
        end for
    end for
    // estimate probabilities
    for all foreign words f do
        for all English words e do
            t(e|f) = count(e|f) / total(f)
        end for
    end for
end while
```

Convergence



e	f	initial	1st it.	2nd it.	3rd it.	...	final
the	das	0.25	0.5	0.6364	0.7479	...	1
book	das	0.25	0.25	0.1818	0.1208	...	0
house	das	0.25	0.25	0.1818	0.1313	...	0
the	buch	0.25	0.25	0.1818	0.1208	...	0
book	buch	0.25	0.5	0.6364	0.7479	...	1
a	buch	0.25	0.25	0.1818	0.1313	...	0
book	ein	0.25	0.5	0.4286	0.3466	...	0
a	ein	0.25	0.5	0.5714	0.6534	...	1
the	haus	0.25	0.5	0.4286	0.3466	...	0
house	haus	0.25	0.5	0.5714	0.6534	...	1

Perplexity

- How well does the model fit the data?

- Perplexity: derived from probability of the training data according to the model

$$\log_2 PP = -\sum_s \log_2 p(\mathbf{e}_s|\mathbf{f}_s)$$

- Example (ϵ=1)

	initial	1st it.	2nd it.	3rd it.	...	final
p(the haus|das haus)	0.0625	0.1875	0.1905	0.1913	...	0.1875
p(the book|das buch)	0.0625	0.1406	0.1790	0.2075	...	0.25
p(a book|ein buch)	0.0625	0.1875	0.1907	0.1913	...	0.1875
perplexity	4095	202.3	153.6	131.6	...	113.8

Higher IBM Models

IBM Model 1	lexical translation
IBM Model 2	adds absolute reordering model
IBM Model 3	adds fertility model
IBM Model 4	relative reordering model
IBM Model 5	fixes deficiency

- Only IBM Model 1 has global maximum
  - training of a higher IBM model builds on previous model

- Compuationally biggest change in Model 3
  - trick to simplify estimation does not work anymore

  $\rightarrow$ exhaustive count collection becomes computationally too expensive
  - sampling over high probability alignments is used instead