Probabilistic Languages

Some Models of Translation

- IBM Models 1-5
- Hidden Markov Model
- Phrase-Based Models

Q: What do all of these things have in common?

“Language”

Definition 1: *Natural* language

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Tends to not be well-defined.

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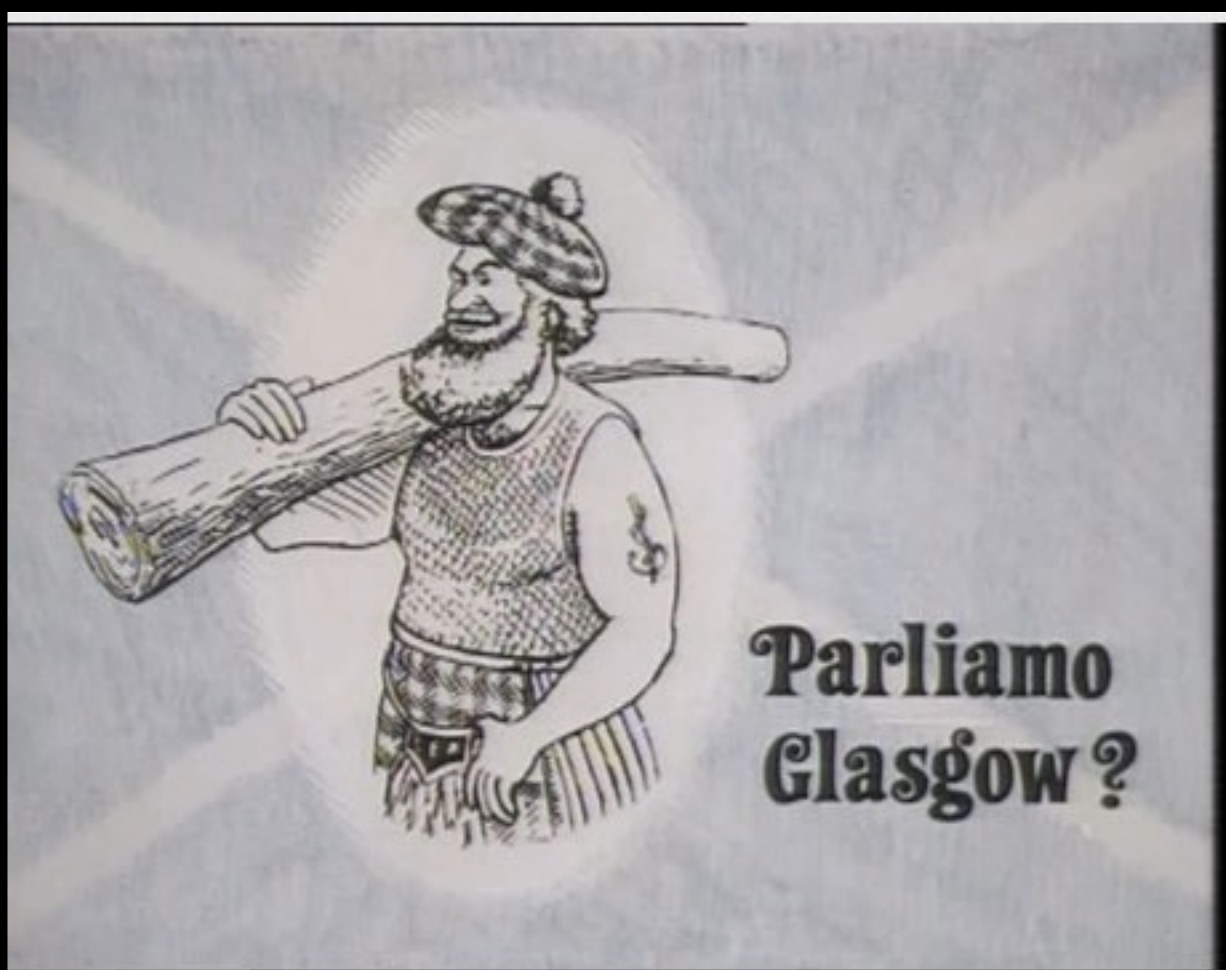

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"Language"

Definition 1: *Formal* language

Well-defined, so that a computer can process it:
a (possibly infinite) set of strings.

- All of the English words in a dictionary.
- All sequences of any length over those words.
- All English sentences with non-zero $p(e \mid f)$ for some French sentence f, according to your model.

Desiderata

- We need efficient algorithms and data structures to:
  - Encode all of the strings in the language.
  - Assign probabilities to all of those strings.
    - Via products such as $p(e)p(f \mid e)$.
  - Find the string with the highest probability.
  - Compute expectations over substrings.
  - Compute mappings between strings.

Regular Languages

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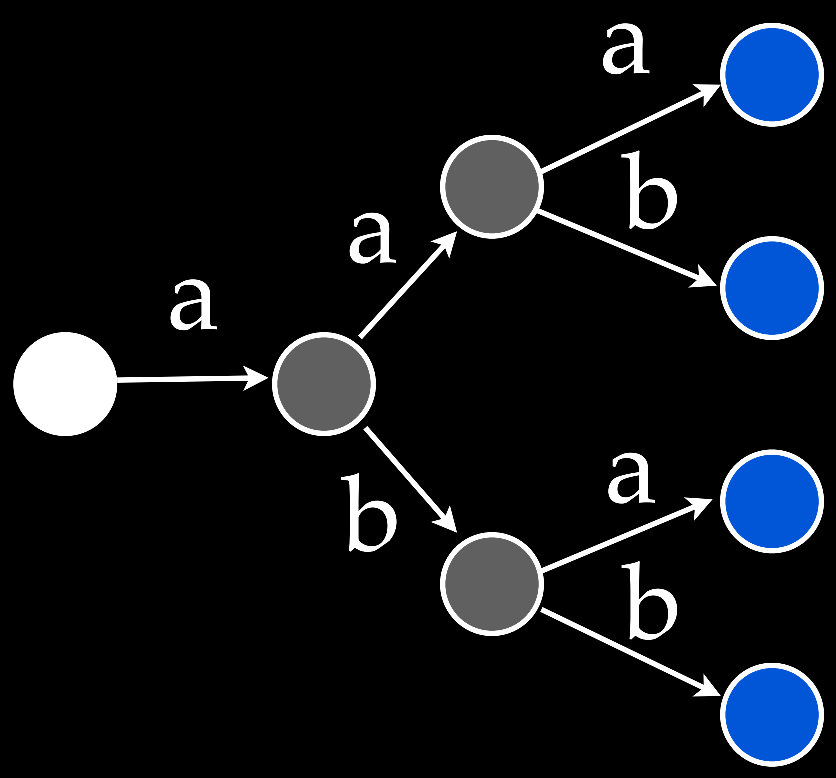

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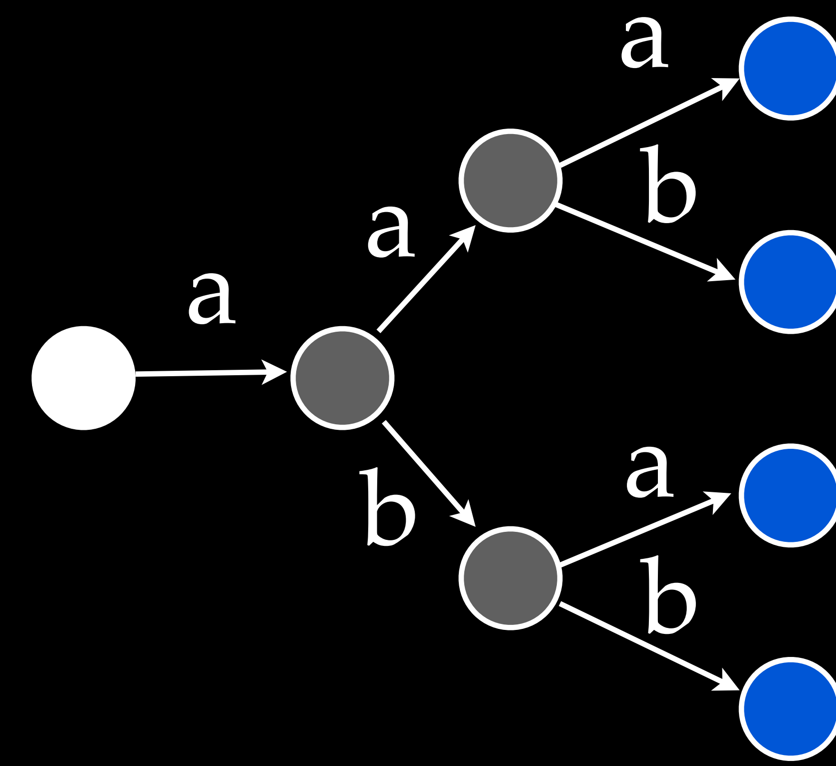

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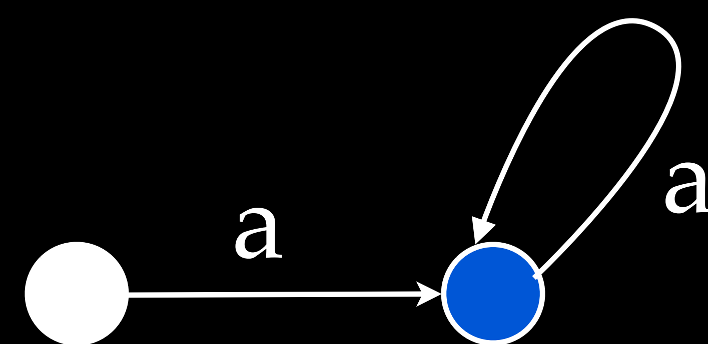

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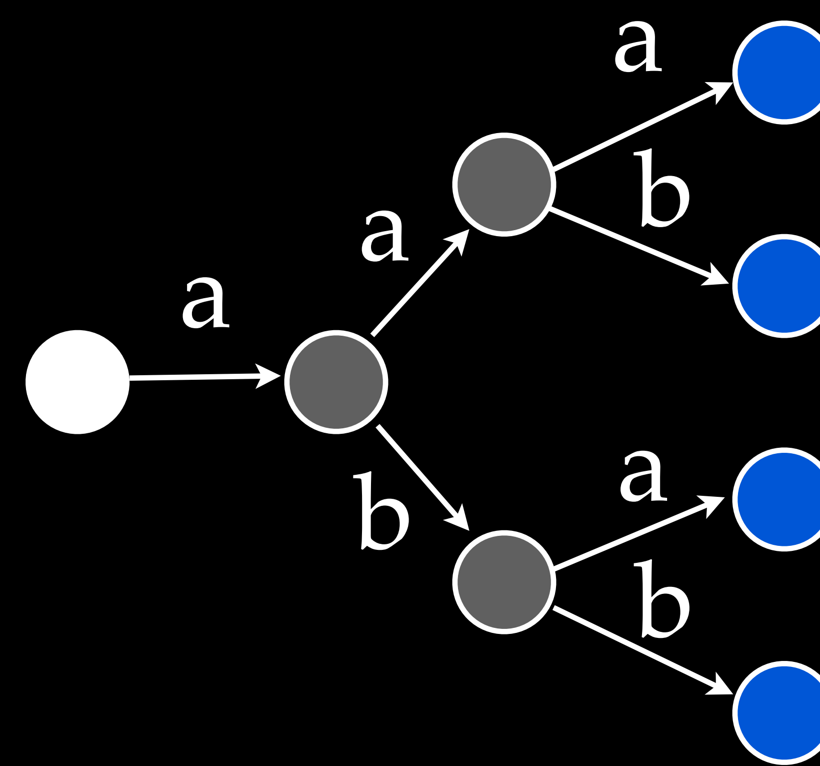

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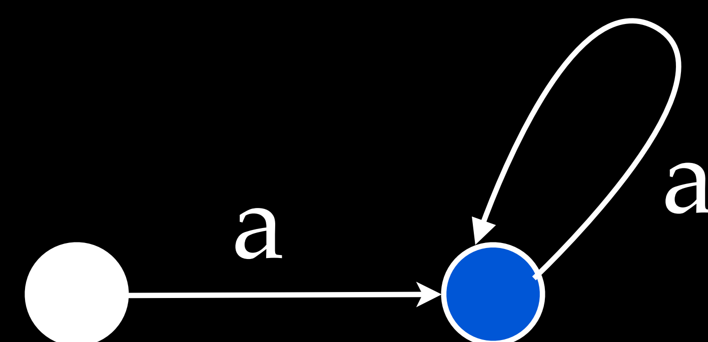

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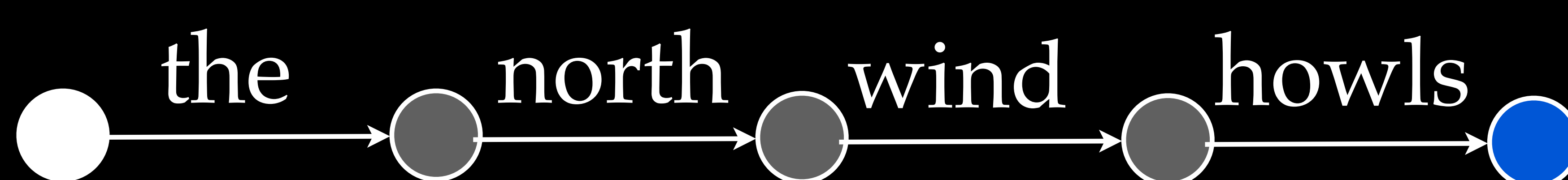

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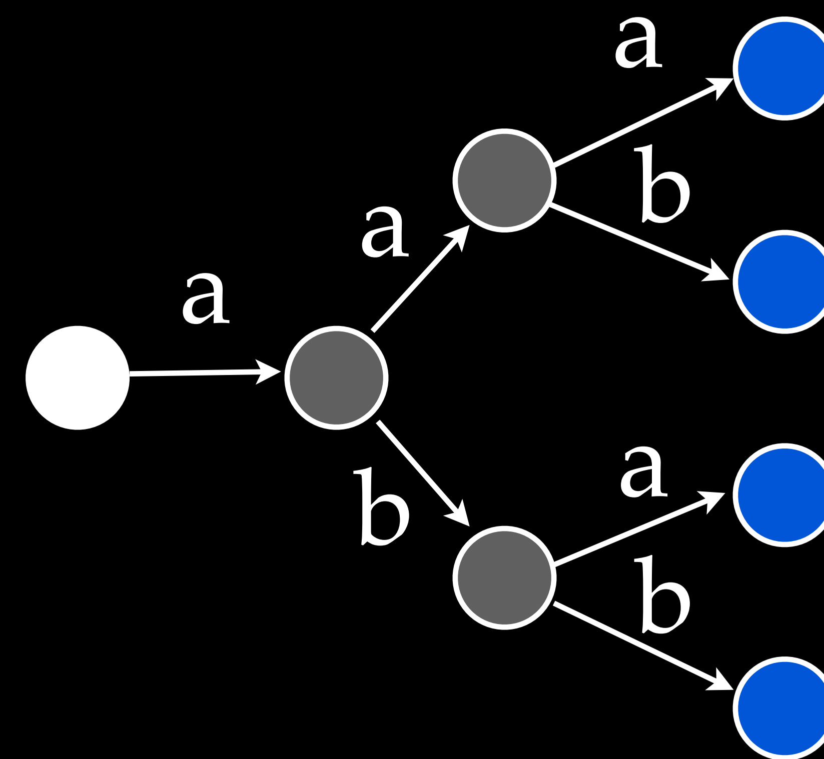

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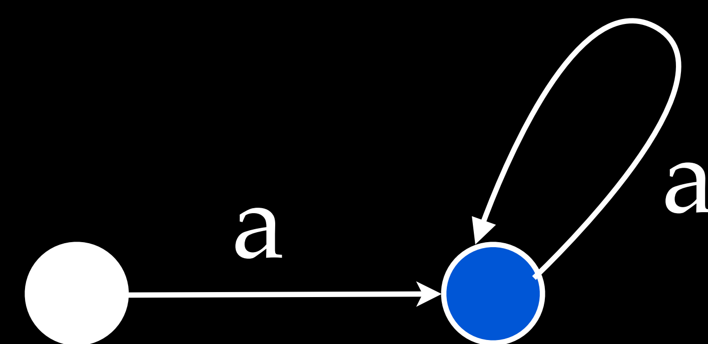

finite-state automata

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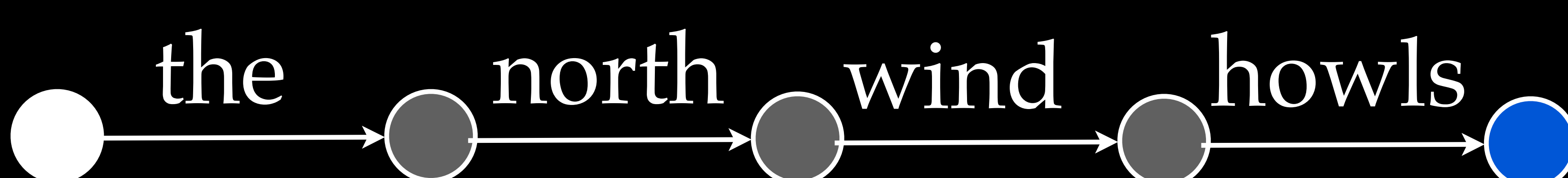

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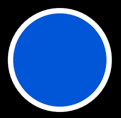
$\{\epsilon\}$ is regular 

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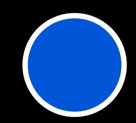

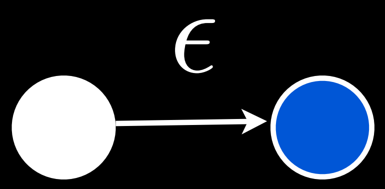

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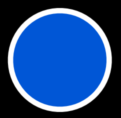

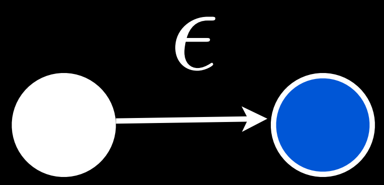

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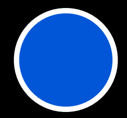

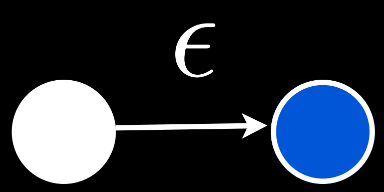

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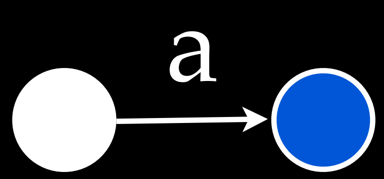

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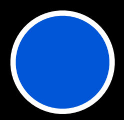

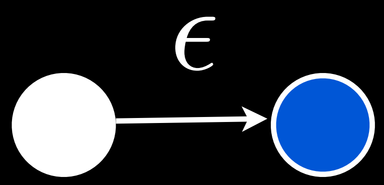

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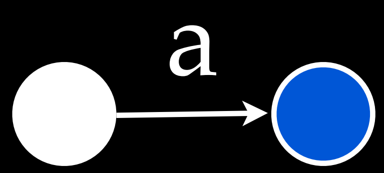

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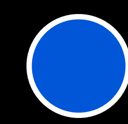

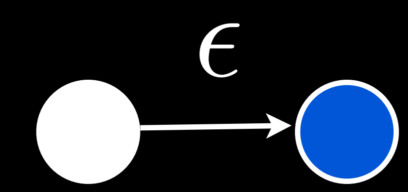

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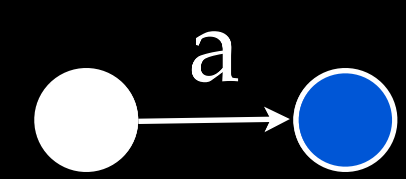

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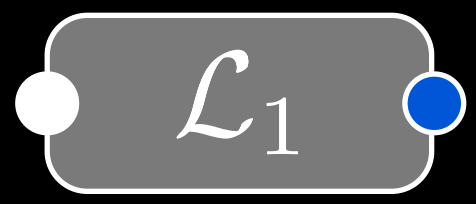

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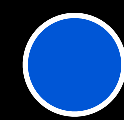

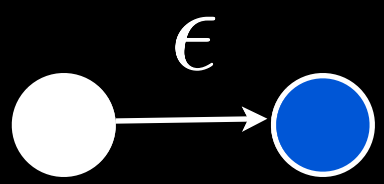

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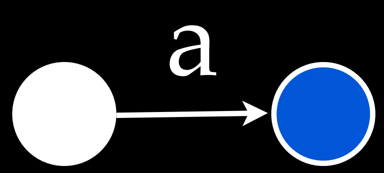

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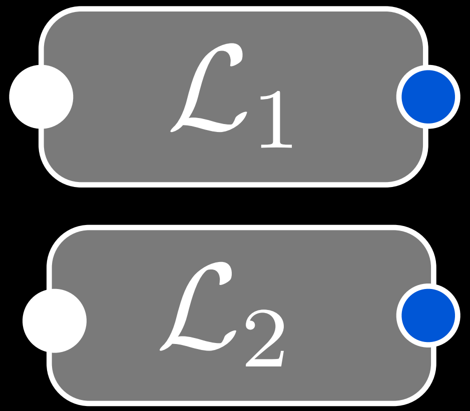

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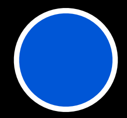

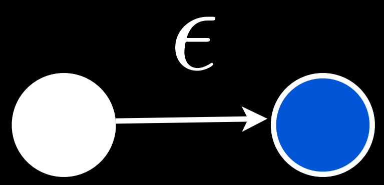

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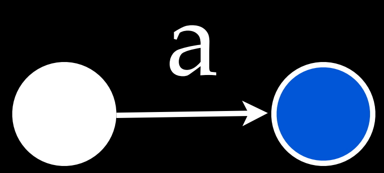

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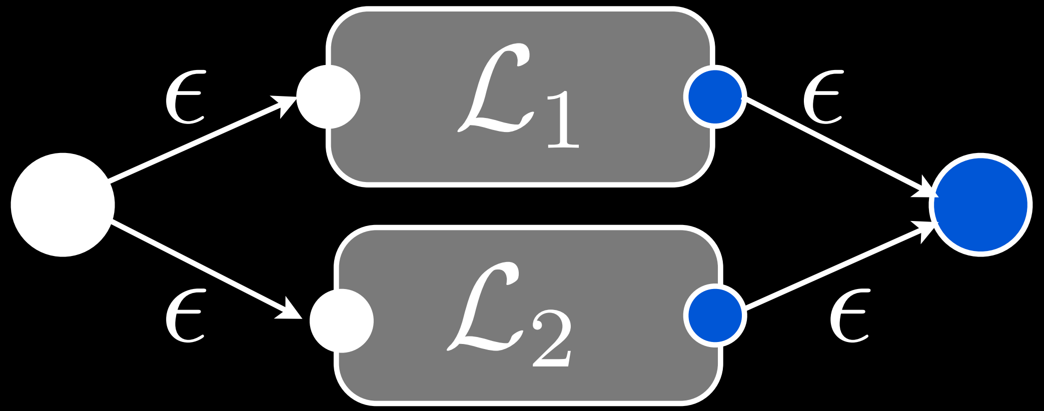

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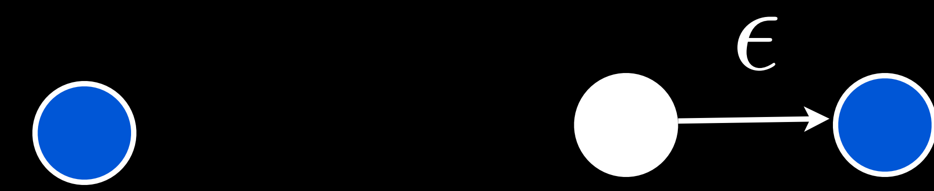

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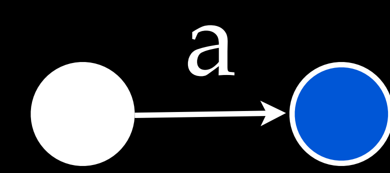

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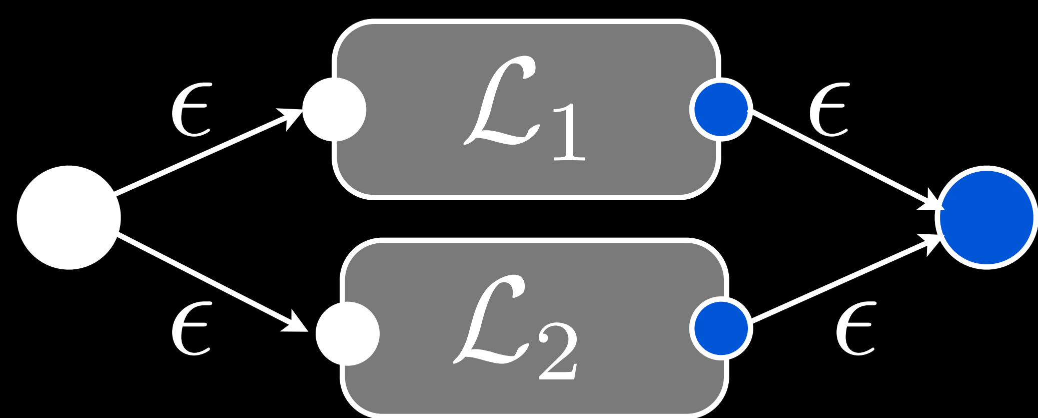

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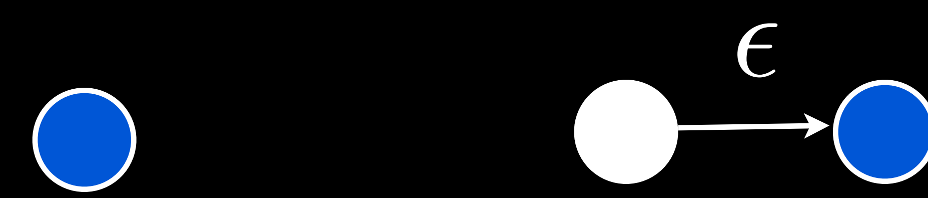

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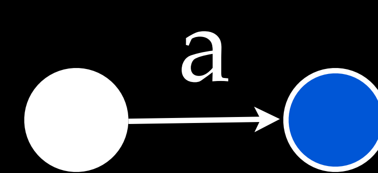

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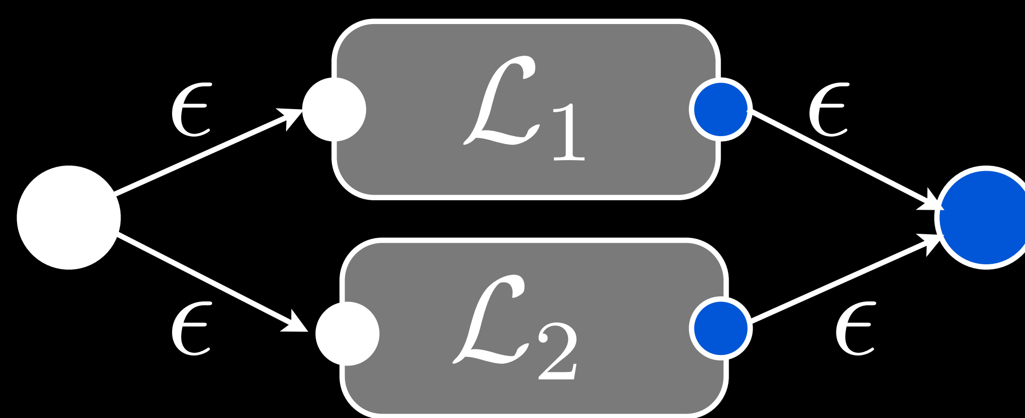

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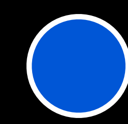

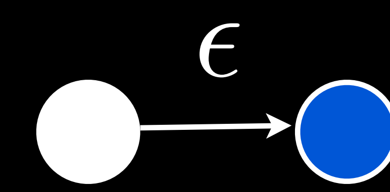

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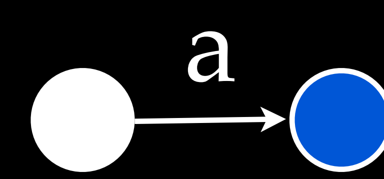

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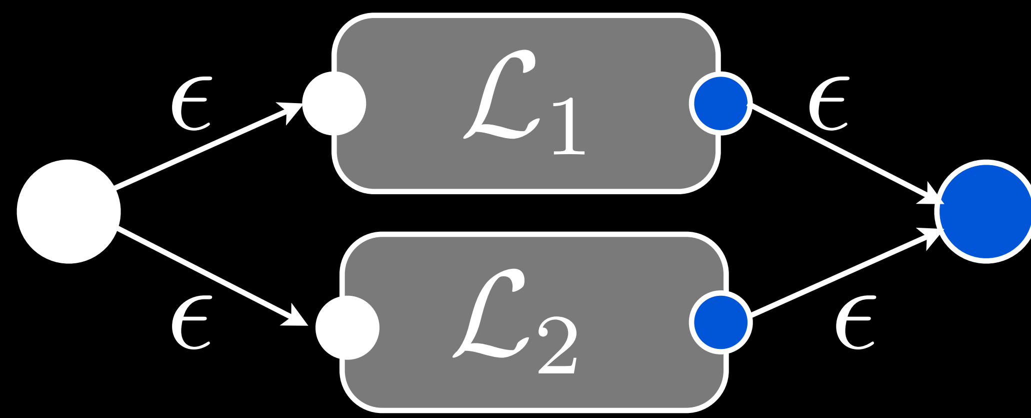

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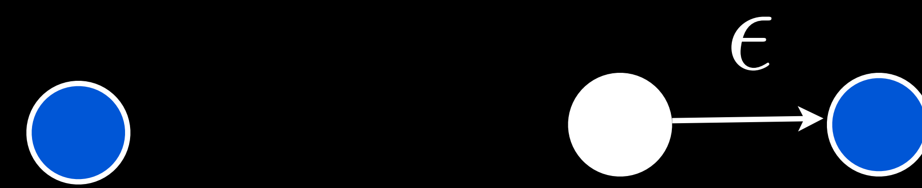

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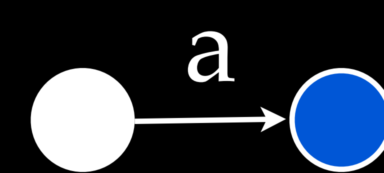

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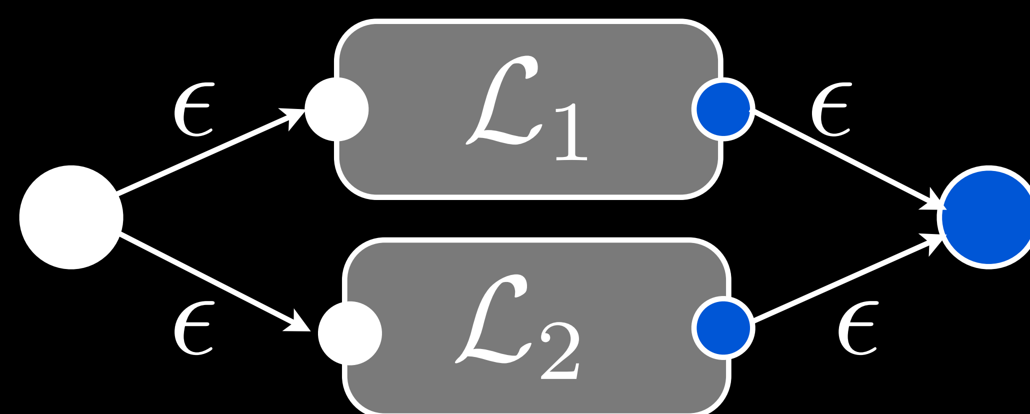

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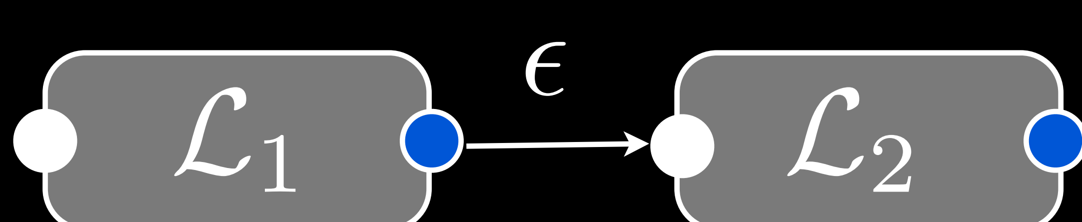

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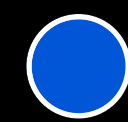

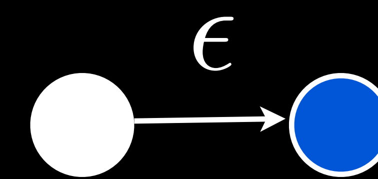

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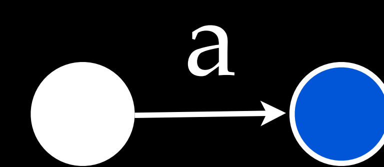

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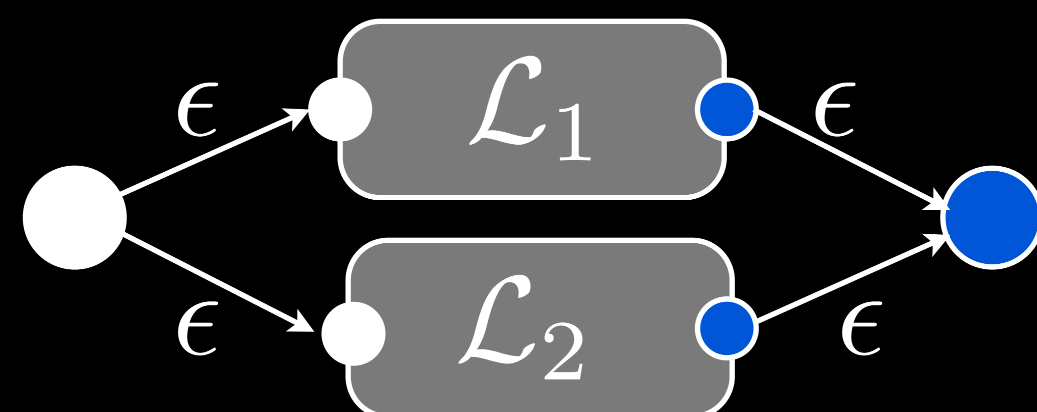

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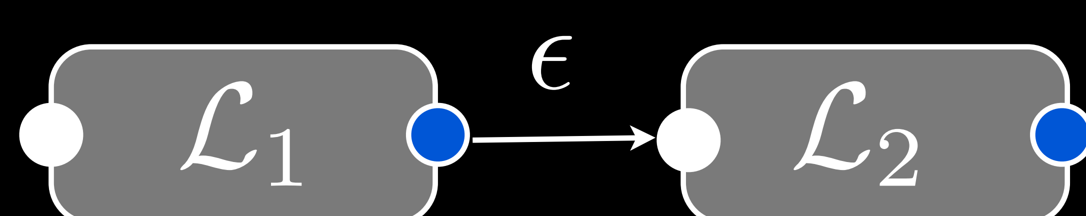

$\mathcal{L}_1^*$

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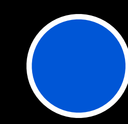

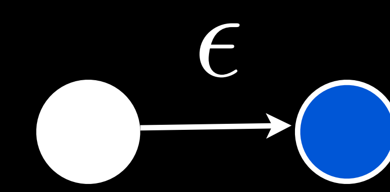

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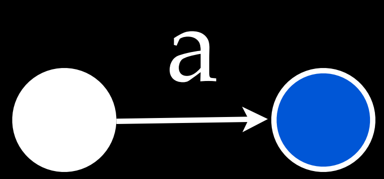

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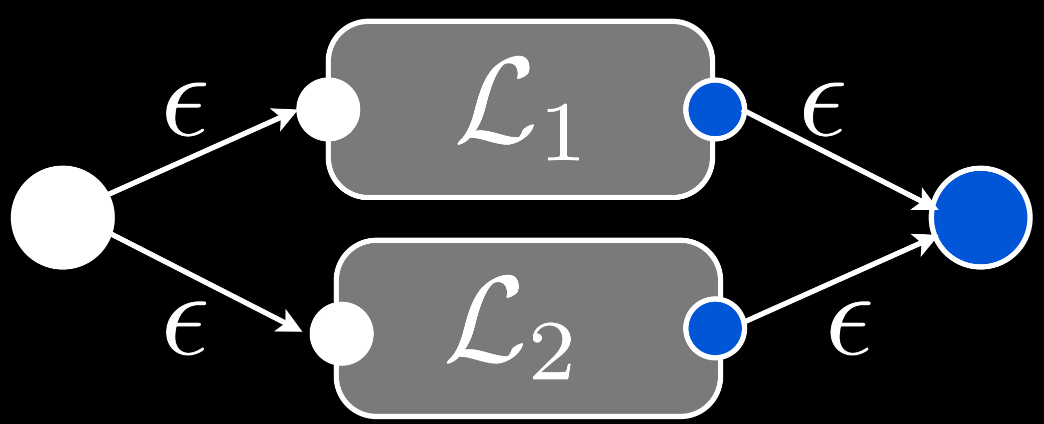

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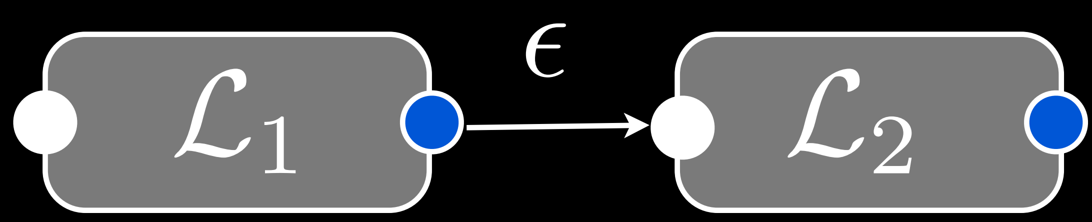

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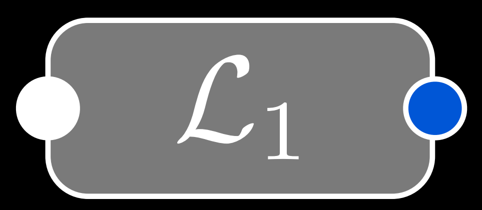

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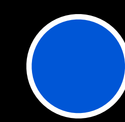

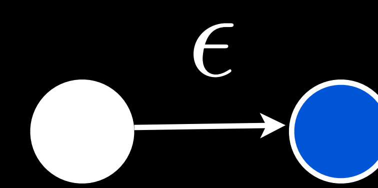

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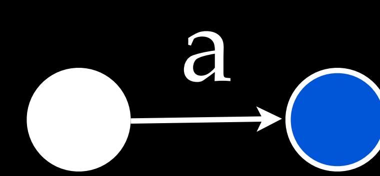
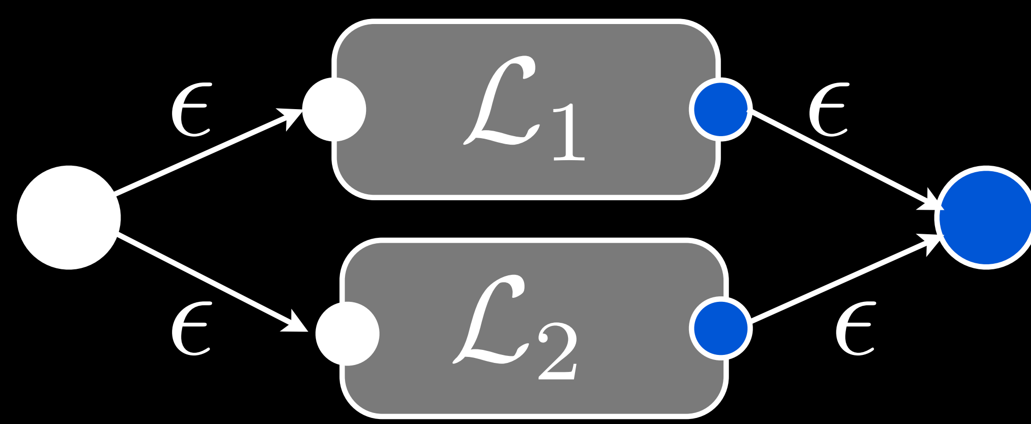
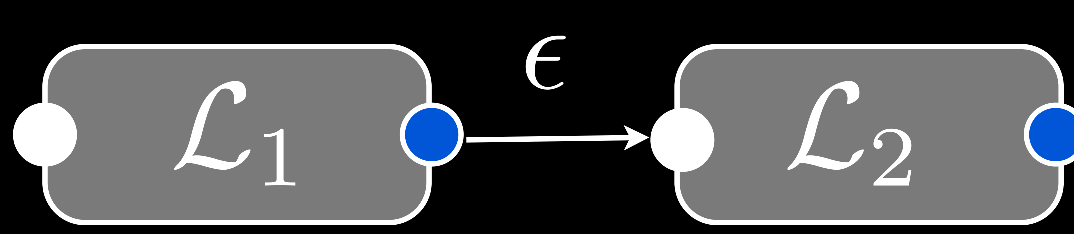


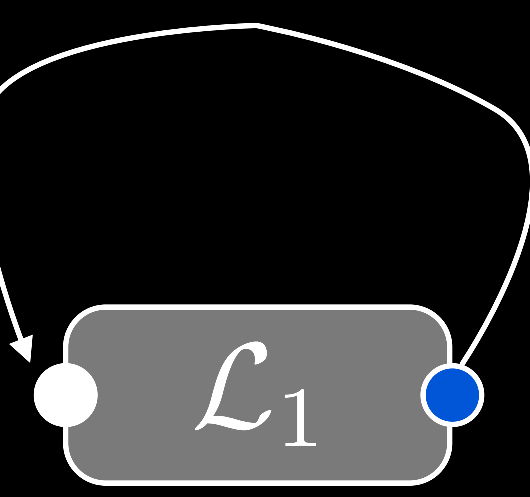
$\mathcal{L}_1 \cup \mathcal{L}_2$ is regular if $\mathcal{L}_1$ and $\mathcal{L}_2$ are regular



$\mathcal{L}_1 \cdot \mathcal{L}_2$



$\mathcal{L}_1^*$



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But not all languages are context-free, either!

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We want a function:

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such that:

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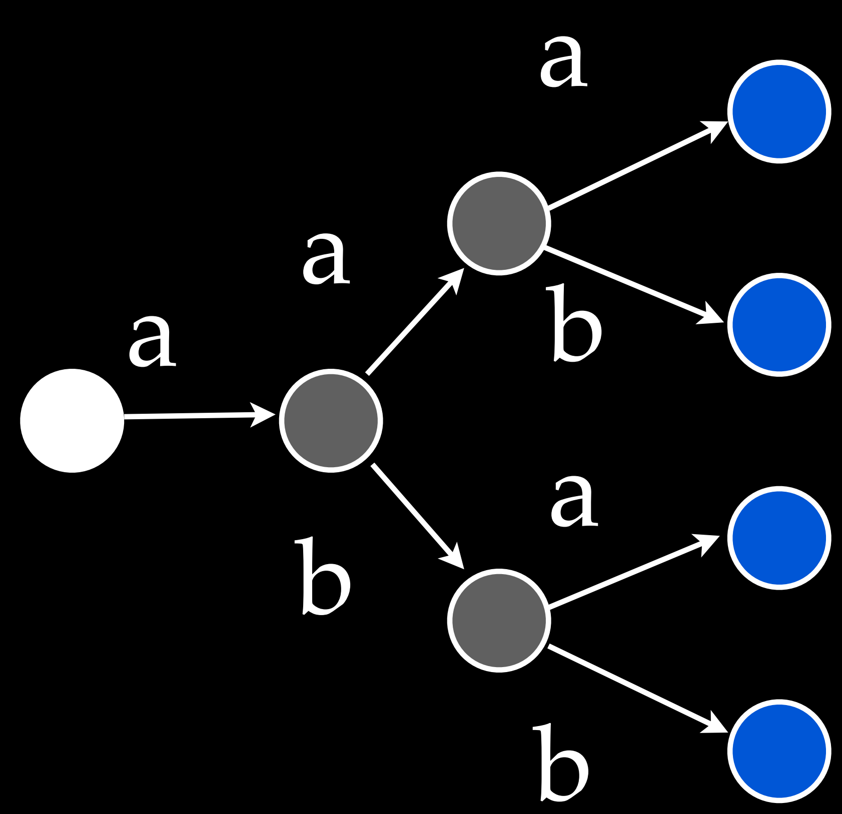

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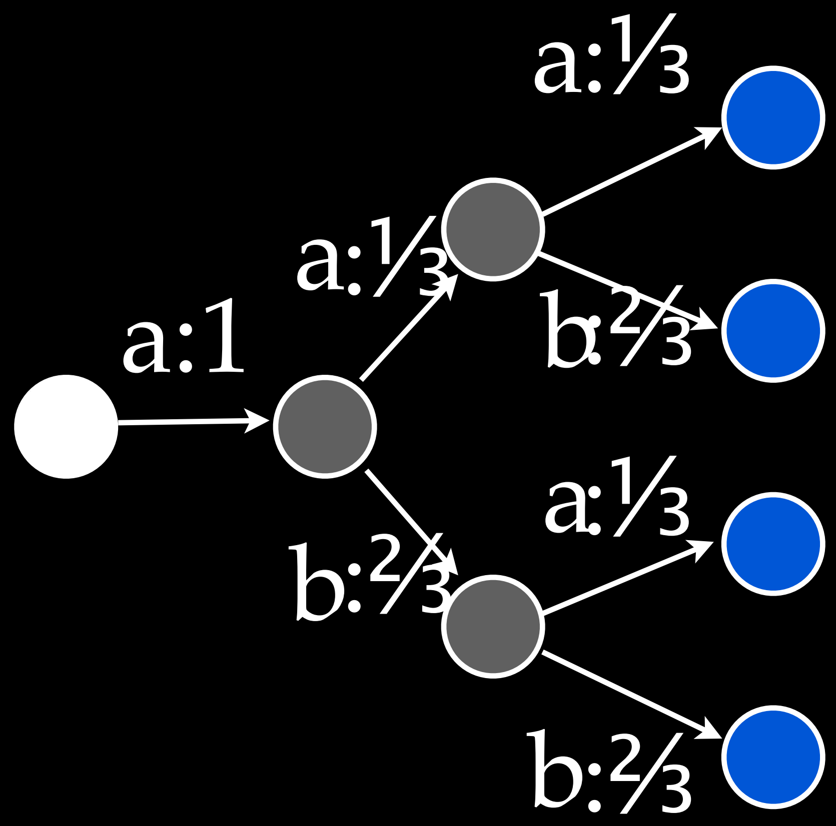
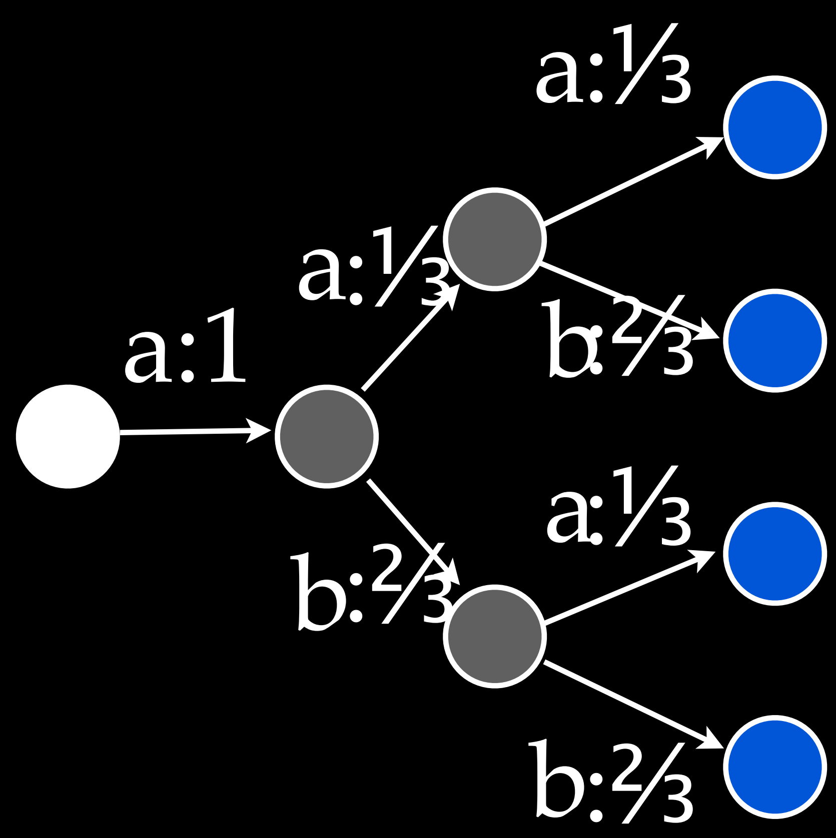


Minimization



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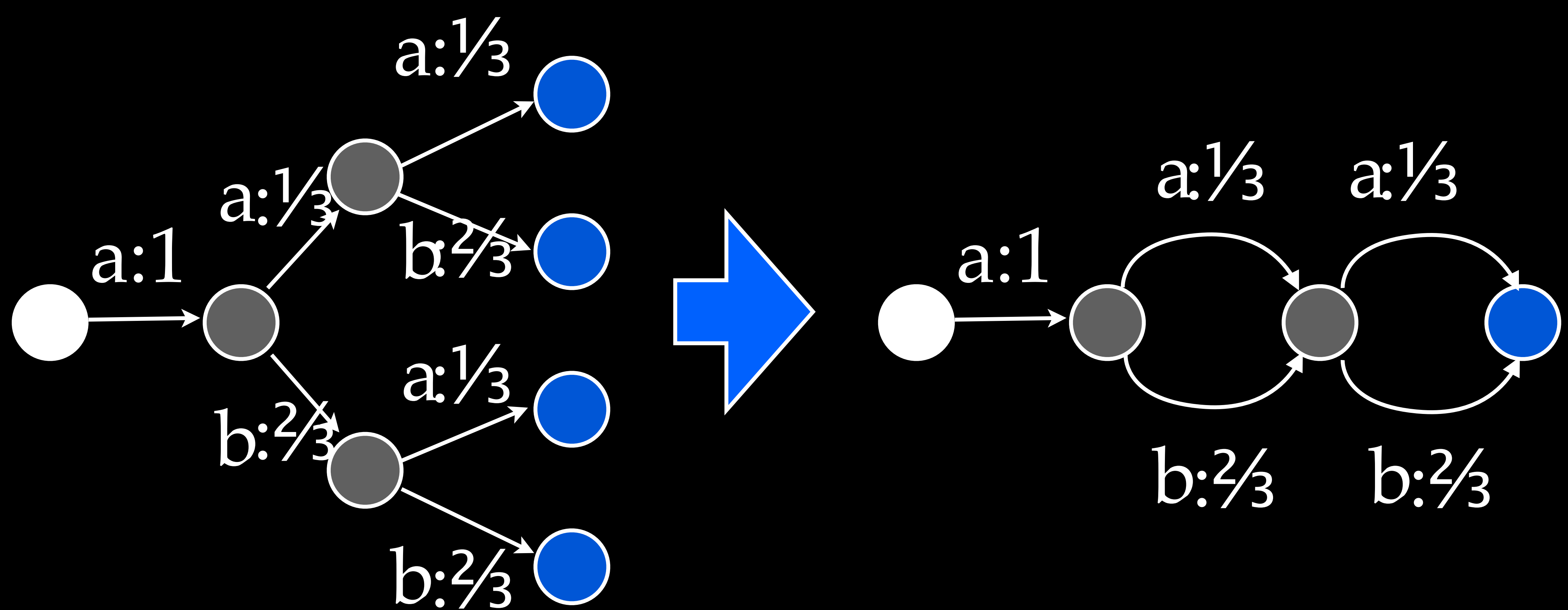

Other Algorithms

- Shortest path (e.g. Dijkstra, A*): most probable
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- Epsilon-removal
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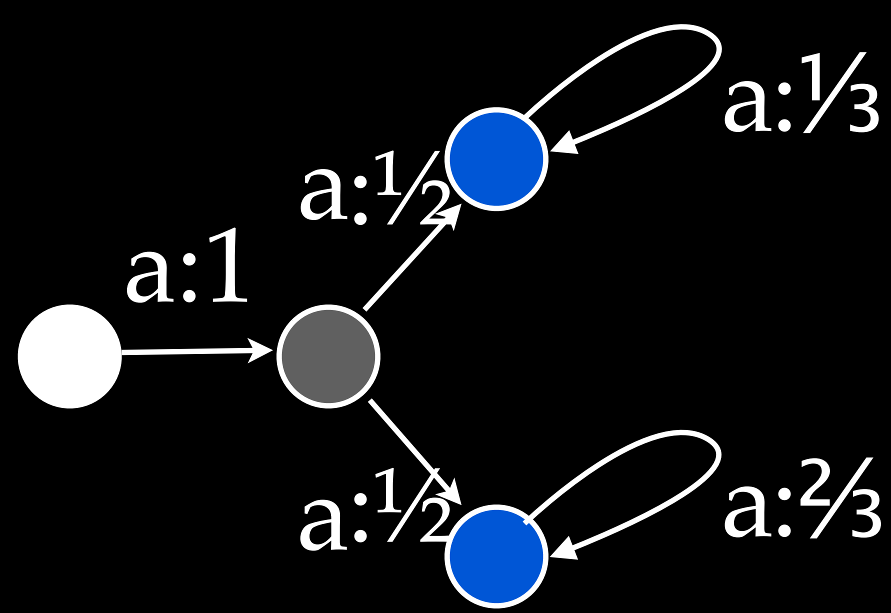

Practical Issues

- OpenFST (openfst.org)
  - Efficient C++ implementation.
  - Used in speech recognition (Google, Kaldi @ JHU)
  - Machine translation (JHU → Cambridge, Google)

Some Models of Translation

- IBM Models 1-5
- Hidden Markov Model
- Phrase-Based Models

Q: What do all of these things have in common?

A: They all define *weighted regular languages* over a set of output sentences. Details Thursday.

Questions

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Are natural languages regular?

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Does it matter for MT if they aren’t?